Signals & Systems
Descriptions of Signals

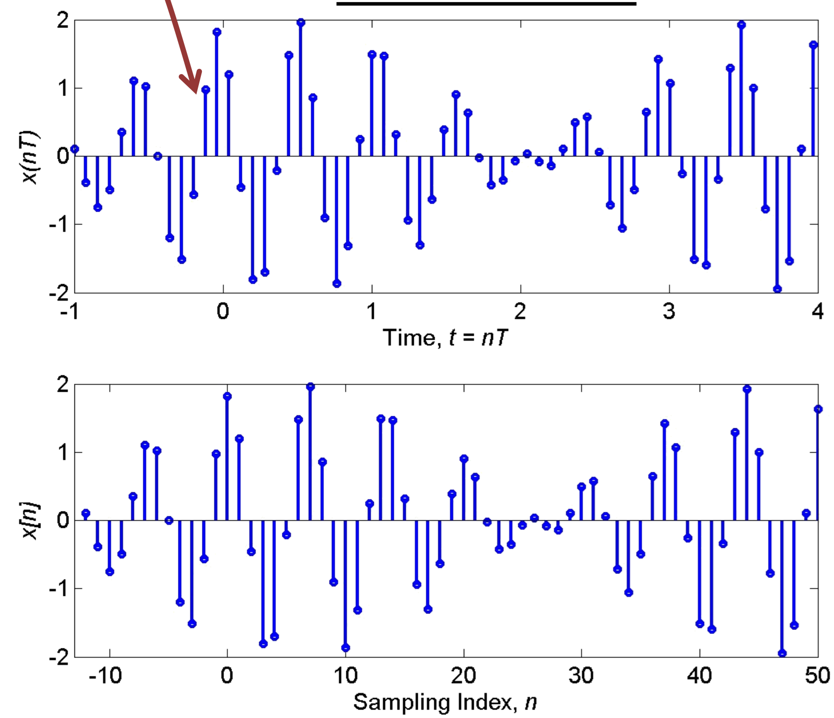
ECE 3793

Continuous-Time vs. Discrete-Time

- Continuous: defined for a continuum of values of *t*
- Discrete: defined only at discrete values of *t* $\quad x(nT) \rightarrow x[n]: \quad -\infty < n < \infty$
  - …or equivalently for integer values of a sampling index
  - The discrete-time signal is *undefined* between these specific values of the independent variable

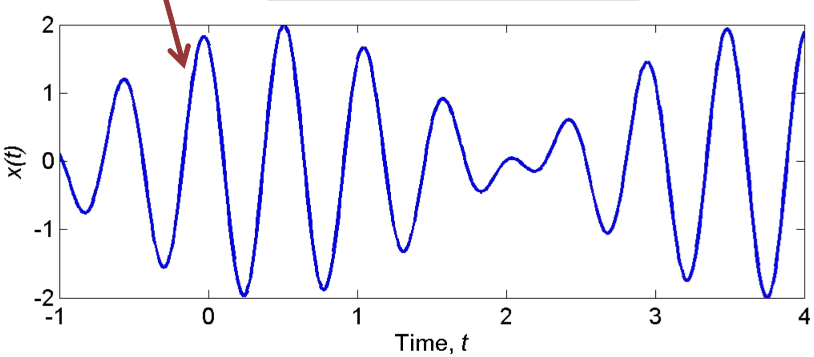
Independent variable *n* must be an integer

Discrete-Time



Independent variable *t* is continuous

Continuous-Time



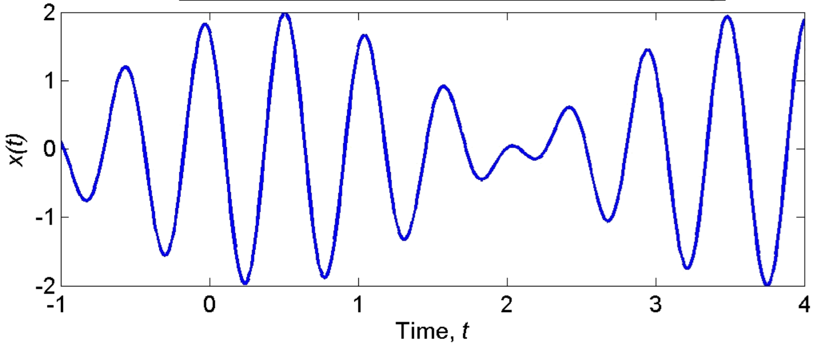
Q: How many unique values of t are there in any finite interval $t \in [a, b]$?

A: ∞

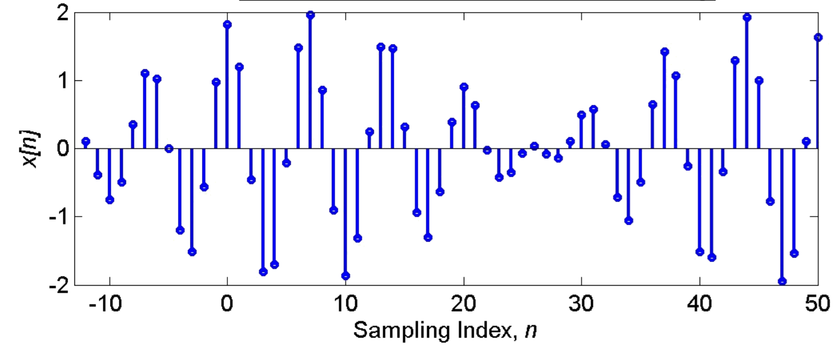
Analog vs. Digital Signals

- Analog: The amplitude can be any value in a continuous range
- Digital: The amplitude must be one of a fine number of values (*quantized*)

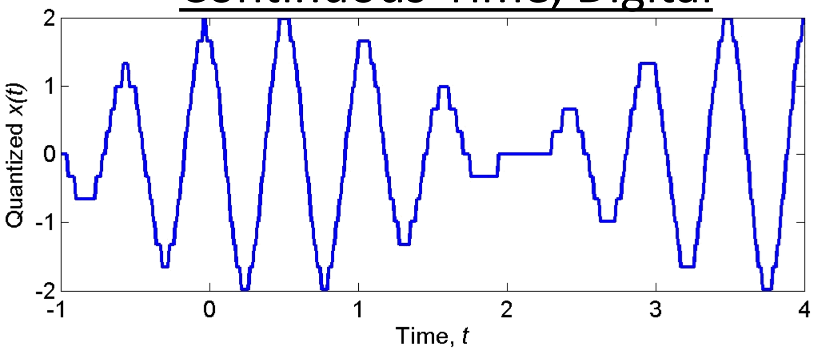
Continuous-Time, Analog



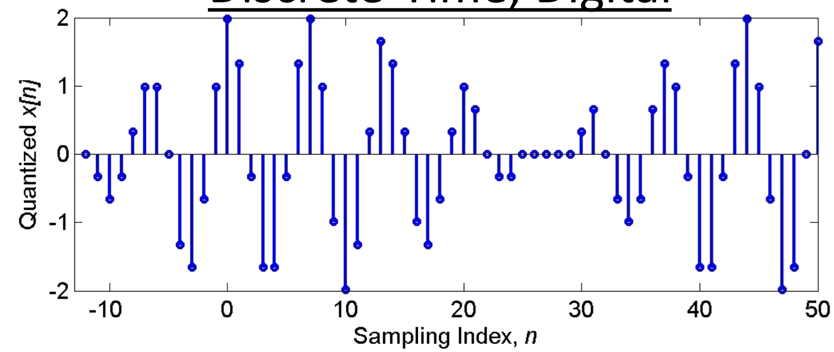
Discrete-Time, Analog



Continuous-Time, Digital



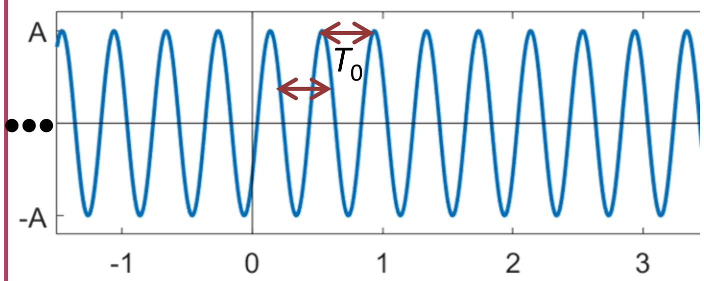
Discrete-Time, Digital



Periodic vs. Aperiodic Signals

- A signal $x(t)$ is periodic if $x(t) = x(t + T_0)$ for all t
  - The smallest value of T_0 for which this is satisfied is the *fundamental period* of $x(t)$
  - Because this must be true for all t (i.e., for all time), periodic signals must start at $t = -\infty$ and continue forever
- Examples: $$x(t) = A\sin(2\pi f_0 t + \theta) \rightarrow x(t + T_0) = A\sin(2\pi f_0 (t + T_0) + \theta)$$

$$= A\sin(2\pi f_0 t + 2\pi f_0 T_0 + \theta)$$



••• ••• If $f_0 T_0$ = integer k, then… $= A\sin(2\pi f_0 t + \theta) = x(t)$

…& smallest $T_0 = 1/f_0$

$$x(t) = A\exp(j2\pi f_0 t + j\theta) \rightarrow \quad x(t + T_0) = A\exp(j2\pi f_0 (t + T_0) + j\theta)$$

$$= A\exp(j2\pi f_0 t + j2\pi f_0 T_0 + j\theta)$$

If $f_0 T_0$ = integer k, then… $= A\exp(j2\pi f_0 t + j\theta) = x(t)$

…& smallest $T_0 = 1/f_0$

- For periodic signals, we have $\int_a^{a+T_0} x(t)\,dt = \int_b^{b+T_0} x(t)\,dt$

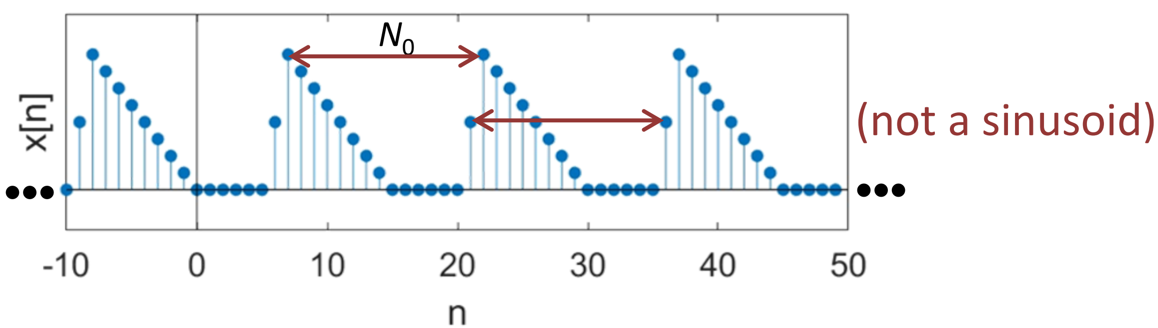
Periodic vs. Aperiodic Signals (DT)

- A discrete-time signal $x[n]$ is periodic if $x[n] = x[n + N_0]$ for all n
  - The smallest value of N_0 for which this is satisfied is the *fundamental period* of $x[n]$
  - Because this must be true for all n (i.e., for all time), DT periodic signals must start at $n = -\infty$ and continue forever
- Examples: $x[n] = A\sin(2\pi f_0 n + \theta) \rightarrow x[n + N_0] = A\sin(2\pi f_0(n + N_0) + \theta)$

$$= A\sin(2\pi f_0 n + 2\pi f_0 N_0 + \theta)$$

If $f_0 N_0$ = integer k, then... $= A\sin(2\pi f_0 n + \theta) = x[n]$

But…there is a quirk with DT sinusoids. Because N_0 must be an integer, it's only possible for $f_0 N_0$ to be an integer if f_0 is rational – i.e., a ratio of integers, $f_0 = k/N_0$. Therefore, DT sinusoids aren't periodic unless freq. follows the form $f_0 = k/N_0$.



- For periodic signals, we have $\sum_{n=a}^{a+N_0-1} x[n] = \sum_{n=b}^{b+N_0-1} x[n]$

How "Big" is a Signal?

- Signal Energy: the area under $x^2(t)$

$$E_x = \int_{-\infty}^{\infty} |x(t)|^2 \, dt$$

Note: Because x(t) can represent anything, & the independent variable doesn't have to be time, not necessarily physical energy (Joules)

- Other metrics are possible, but energy is mathematically tractable

- Example: Find the energy in $x(t) = \mathrm{rect}(t)$, where

$$rect(t) \triangleq \begin{cases} 1 & |t| < \frac{1}{2} \\ 0 & \text{else} \end{cases}$$

- Example: Find the energy in $x(t) = A\cos(\omega t)$

- Discrete Form: $E_x = \sum_{n=-\infty}^{\infty} |x[n]|^2$

How "Big" is a Signal?

- When E = ∞, not too useful, so…
- Signal power: $$P_x = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 \, dt$$

(power is time-averaged energy, or the rate of energy)

- Find P_x for $x(t) = \mathrm{rect}(t)$
- Find P_x for $x(t) = A\cos(\omega t)$
- Power is the average squared magnitude: mean-squared value of $x(t)$
- The square root of power, $\sqrt{P_x}$, is the root-mean-square (RMS)
- P_x for periodic signals can be obtained by averaging energy over 1 period
- Discrete version: $$P_x = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x[n]|^2$$

Energy vs. Power Signals

- The continuous-time signal x(t) (or discrete-time signal x[n]) is an "energy signal" if and only if:
  - $0 < E_x < \infty$, such that $P_x = 0$
  - Signals that "die out"/decay to zero tend to be energy signals
    - Finite-duration (and finite amplitude) signals will always be energy signals!
- The continuous-time signal x(t) (or discrete-time signal x[n]) is a "power signal" if and only if:
  - $0 < P_x < \infty$, such that $E_x = \infty$
  - Signals that don't decay tend to be power signals
    - Periodic signals will always be power signals!

Misc. Signal Descriptions

- Causal/Non-Causal (will be more important when we relate to systems)
  - If $x(t) = 0$ for $t < 0$, then causal signal
  - If $x(t) = 0$ for $t > 0$, the anti-causal signal
  - Otherwise, non-causal
- Right-Sided/Left-Sided/Two-Sided/Finite Duration
  - If $x(t) = 0$ for $t < t_0$, and then continues forever to $t = \infty$, then "right-sided"
  - If $x(t) = 0$ for $t > t_0$, but continues forever to $t = -\infty$, then "left-sided"
  - If $x(t)$ continues forever in both directions to $t = -\infty$ and $t = \infty$, the "two-sided"
  - If $x(t)$ only lasts for a finite amount of time, the "finite duration"
- Deterministic vs. Random (we won't be concerned with this one)
  - Deterministic: signal value is completely known at all times/indexes/etc…
    - Examples: $x(t) = e^{-t}$ or $x(n, z) = e^{-2n}\delta(z - 5)$
  - Random: value of the signal must be described probabilistically
    - Using average values, correlations, etc.
    - Example: noise
- Symmetries
  - Even symmetry: $x(t) = x(-t)$ (time reversal produces the same signal)
  - Odd symmetry: $x(t) = -x(-t)$ (time reversal produces a sign-reversed signal)
    - Note #1: for odd signals, we must have $x(0) = 0$ or $x[0] = 0$
    - Note #2: Any signal can be broken up into the sum of an even signal and an odd signal
  - Conjugate symmetry: $x(t) = x^*(-t)$